# COUNTER-FLOW WORT CHILLLER

## 1.0 Intro

The counter-flow chiller consists of a pair of channels which are thermally connected. The substance to be cooled (wort) flows through one and the coolant (water)through the other. The directions of flow are opposite to one another such that the exiting wort is in contact with the entering coolant at which point the coolant is coldest. This insures that the exiting wort is as cool as possible. We assume that the geometry of the chiller is unchanging along the length of the chiller such that the thermal conductance between the two channels per unit length is a constant. We say nothing about the actual geometry because any of a several geometries are possible and which is chosen is immaterial to the analysis subject to the constraint just named.

Define an $x$ axis with the origin at the point where hot wort enters the chiller. In passing throuh the chiller the wort temperature drops to a minimum at the distal end, $x = L$. Its temperature along the chiller is described by the function $T_w(x)$ .Similarly, $T_c(x)$ describes the temperature of the coolant as a function of position within the chiller.

Consider an incremental length $dx$ located at $x$ with respect to the origin which we define as the point at which the wort enters and the coolant exits. The temperatures of wort and coolant at this point are given by $T_w(x)$ and $T_c(x)$ respectively. Heat flows from the wort to the coolant at $x$ at rate

$$\frac{dH}{dt} = WG[T_w(x) - T_c(x)]dx \tag{1.1}$$

through the area $Wdx$ i.e. the area of contact between the two channels where $G$ is the thermal conductivity per unit area. In time $dt$ a volume of fluid $Fdt$, where $F$ is a flow rate (e.g. lpm,gph) passes through the volume defined by $dx$ where it picks up (or loses) heat $dH$. Letting $\rho$ represent the fluid density and $C_p$ the fluid specific heat (at constant pressure) we have

$$\frac{dM}{dt} = F\rho C_p \tag{1.2}$$

as the rate at which *thermal* mass passes through the volume. The heat rise or drop in this mass is

$$dT = \frac{dH}{dM} = \frac{WG[T_w(x) - T_c(x)]dtdx}{F\rho C_p dt} \tag{1.3}$$

so that

$$\frac{dT}{dx} = \frac{WG[T_w(x) - T_c(x)]}{F\rho C_p} \tag{1.4}$$

describes the rate of change of temperature with $x$. Subscripting for wort and coolant respectively gives the pair of differential equations:

$$\frac{dT_w}{dx} = \frac{WG[T_w(x) - T_c(x)]}{F_w\rho_w C_{p,w}} = -k_w[T_w(x) - T_c(x)] \tag{1.5}$$

$$\frac{dT_c}{dx} = \frac{WG[T_w(x) - T_c(x)]}{F_c\rho_c C_{p,c}} = -k_c[T_w(x) - T_c(x)] \tag{1.6}$$

Their solution describes the behaviour of the chiller. Note that in defining the constants

$$k_w = \frac{WG}{F_w\rho_w C_{p,w}} \tag{1.7}$$

$$k_c = \frac{WG}{F_c\rho_c C_{p,c}} \tag{1.8}$$

and writing Equations (1.5) and (1.6) as we did (with minus signs in from of $k_w$ and $k_c$) we make both derivatives negative. Thus the temperature of both wort and coolant decrease as we move away from the wort inlet/coolent outlet) as $T_w(x) \geq T_c(x) \forall 0 \leq x \leq L$ where $x = L$ represents the wort outlet/coolant inlet position.

Subtracting Equation(1.6) from (1.5) gives

$$\frac{d}{dx}[T_w - T_c] = (k_c - k_w)[T_w - T_c] \tag{1.9}$$

i.e the simplest form of differential equation whose solution is

$$[T_w(x) - T_c(x)] = [T_w(0) - T_c(0)]e^{ax} + \Delta T_\infty \tag{1.10}$$

i.e. and exponential plus a constant. Substitution of this solution into Equation (1.9) and setting makes it clear that:

(1.11)

A chiller will always be operated with a coolant thermal mass flow ($F_c\rho_c C_{p,c}$) larger than that of the wort ($F_w\rho_w C_{p,w}$) as failure to do so would result in ineffective wort cooling. Thus $k_c < k_w$ and $a < 0$. We therefore, as a matter of convenience, define

$$\alpha \equiv -a = k_w - k_c > 0 \tag{1.12}$$

and write

$$[T_w(x) - T_c(x)] = [T_w(0) - T_c(0)]e^{-ax} + \Delta T_\infty \tag{1.13}$$

The only reason for doing this is that it may be easier for readers to appreciate that the temperature difference between wort and coolant is decreasing with increasing $x$ provided that $\alpha > 0$ which requires $k_w < k_c$. If it is and we set $x = \infty$ we see that $\Delta T_\infty = 0$. Thsu $\Delta T_\infty$ is the temperature difference at the coolant input end of an infinitely long chiller. It is not hard to see that $\Delta T_\infty = 0$.

If $k_w = k_c$ then $\alpha = 0$ and the temperature difference between wort and coolant will not change over the length of the chiller. We cannot, from Equation (1.10) alone determine what that difference may be but it will be constant. Physically the reason for this is that the heat lost by each unit mass of wort results in a temperature drop which is exactly equal to the temperature rise experienced by an equal mass of coolant. Since the flow rates are the same, equal masses of coolant and wort exit the chiller in a given time.

In the event $k_w > k_c$, as would be the case if the wort flows faster than the coolant, the temperature difference between wort and coolant would increase over the length of the chiller. In this situation we would relable ends of the chiller making the wort outlet/coolant inlet end correspond to $x = 0$ and measure $x$ towards the wort inlet/coolant outlet. This would enable us to continue to use Equation (1.13). For everything from this point forward, however, we will impose the restriction that the thermal mass flow of the coolant be greater than the that of the wort ($F_c\rho_c C_{p,c} > F_w\rho_w C_{p,w}$) because operating it in violation of that condition would not result in effective wort cooling.

Given the foregoing it should not be surprising that the solutions to Equation (1.5) and Equation (1.6) are of the form:

$$T_w(x) = T_\infty + T_w' e^{-\alpha x} \tag{1.14}$$

$$T_c(x) = T_\infty + T_c' e^{-\alpha x} \tag{1.15}$$

Thus $T_\infty$ is the temperature of the wort exiting a chiller that is infinitely long and as such has had infinite time to exchange heat with the coolant and thus must be at the same temperature as the coolant at the coolant entry point. $T_w'$ is then the temperature rise of the incoming wort with respect to the incoming coolant temperature and $T_c'$ the temperature rise of the exiting coolant with respect to the entering coolant. $T_w'$ and $T_c'$ have physical relevence only for a fictional cooler that is infinitely long but are, nevertheless, parameters that are needed to describe finite length chillers describable by $k_w$ and $k_c$ neither of which, note, depends on the length of the chiller but rather on its "per unit length" characteristics.

It is clear from Equation (1.5) and Equation (1.6) that

$$\frac{dT_c}{dx} = \frac{k_c}{k_w}\frac{dT_w}{dx} \tag{1.16}$$

Differentiating Equation (1.15) gives

$$\frac{dT_c}{dx} = -\alpha T_c' e^{-\alpha x} = \frac{k_c}{k_w}\frac{dT_w}{dx} = -\alpha T_w' \frac{k_c}{k_w} e^{-\alpha x} \tag{1.17}$$

from which we see that

$$T_c' = T_w' \frac{k_c}{k_w} \tag{1.18}$$

Thus we have

$$T_w(x) = T_\infty + T_w' e^{-\alpha x} \tag{1.19}$$

and

$$T_c(x) = T_\infty + T_w' \frac{k_c}{k_w} e^{-\alpha x} \tag{1.20}$$

The boundary conditions for this problem are given by $T_w(0)$ i.e. the temperature of the incoming wort and $T_c(L)$ which is the temperature of the incoming coolant. Insert these into the last two equations and subtract. This gives

$$T_w(0) - T_c(L) = T_w' - T_w'\frac{k_c}{k_w}e^{-\alpha L} \tag{1.21}$$

which can be solved for $T_w'$:

$$T_w' = \frac{T_w(0) - T_c(L)}{\left(1 - \frac{k_c}{k_w}e^{-\alpha L}\right)} \tag{1.22}$$

Putting this into Equation (1.19) for $x = 0$ gives

$$T_\infty = T_w(0) - T_w' \tag{1.23}$$

$T_c'$ is found from Equation (1.18) and with it in hand we have completed the solution of Equation (1.14) and Equation (1.15) in terms of the boundary condition temperatures and the constants $k_w$ and $k_c$.

Substituting Equation (1.22) and Equation (1.23) into Equation (1.19) and rearranging terms gives

$$T_w(x) = T_w(0) + (T_c(L) - T_w(0))\frac{(1 - e^{-\alpha x})}{1 - \frac{k_c}{k_w}e^{-\alpha L}} \tag{1.24}$$

At $x = L$ we find, after rearrangement that

$$\frac{T_w(0) - T_w(L)}{T_w(0) - T_c(L)} = \frac{1 - e^{-\alpha L}}{1 - \frac{k_c}{k_w}e^{-\alpha L}} \tag{1.25}$$

whose right hand side really defines the performance of the chiller. The value of Equation (1.24) is the drop in the temperature of the wort as it passes through the chiller normalized by the difference between the inlet wort and coolant temperatures. When its value reaches 1 the wort exit temperature is equal to the coolant inlet temperature i.e. the chiller is doing the best it can possibly do.

Thus we are inclined to call Equation (1.25) the efficiency of the chiller defined by:

$$\eta = \frac{1 - e^{-\alpha L}}{1 - \frac{k_c}{k_w} e^{-\alpha L}} \tag{1.26}$$

and then

$$T_w(L) = T_w(0) + (T_c(L) - T_w(0))\eta \tag{1.27}$$

quickly gives the wort output temperature as a function of this efficiency, the wort inlet temperature and the difference between the wort inlet temperature and the coolant inlet temperature.

As long as the device is functioning as a chiller i.e. $\alpha > 0$, it is clear that $k_w > k_c$ so that $\frac{k_c}{k_w} < 1$ which implies that

$$\lim_{L \to \infty} \eta = 1 \tag{1.28}$$

In other words, an infinitely long chiller in which the coolant flows faster than the wort is 100% efficient: wort leaves the chiller at the coolant entry temperature.

We will want to look at the limit of $\eta$ as $k_c \to k_w$ (equivalent to $\alpha \to 0$) for at first blush it would seem that this condition would result in $\eta = 1$ irrespective of the length of the chiller. Let $k_c = k_w - \alpha$ then the limit as $\alpha \to 0$ is

$$\lim_{\alpha \to 0} \eta = \lim_{k_c \to k_w} \eta = \lim_{\alpha \to 0} \frac{1 - e^{-\alpha L}}{1 - \frac{k_w - \alpha}{k_w} e^{-\alpha L}} = \lim_{\alpha \to 0} \frac{\alpha L}{1 - \left(1 - \frac{\alpha}{k_w}\right)(1 - \alpha L)} = \lim_{\alpha \to 0} \frac{\alpha L}{\alpha L + \frac{\alpha}{k_w} + \alpha^2 L} = \frac{k_w L}{k_w L + 1} \tag{1.29}$$

It is also of interest to look at what happens to $\eta$ as $k_c$ becomes very small with respect to $k_w$. This would be the case where the coolant flow is very high or, conversely where wort flow is very low. In either case $\frac{k_c}{k_w} \to 0$. Then

$$\lim_{\frac{k_c}{k_w} \to 0} \eta = \lim_{\frac{k_c}{k_w} \to 0} \frac{1 - e^{-(k_w - k_c)L}}{1 - \frac{k_c}{k_w} e^{-(k_w - k_c)L}} = 1 - e^{-k_w L} \tag{1.30}$$

This says that as the wort flow gets small ($k_w$ gets large) even a short chiller approaches 100% efficiency.

Of less practical but perhaps equal academic interest is the outlet temperature of the coolant:

$$T_c(0) = T_\infty + T_w{}'\frac{k_c}{k_w} = T_w(0) + \frac{(T_c(L) - T_w(0))}{1 - \frac{k_c}{k_w}e^{-\alpha L}}\left(1 - \frac{k_c}{k_w}\right) \tag{1.31}$$

$k_w$ is defined in Equation (1.5) where it is seen to be

$$k_w = \frac{WG}{F_w \rho_w S_w} \tag{1.32}$$

and similarly for $k_c$ so that their ratio is:

$$\frac{k_c}{k_w} = \frac{F_w \rho_w S_w}{F_c \rho_c S_c} \tag{1.33}$$

which leads to

$$\lim_{F_c \to \infty} T_c(0) = T_c(L) \tag{1.34}$$

This agrees with simple reasoning which tells us that if the coolant volume passing through the chiller in a unit time is very very large that volume's temperature rise will be very very small even though a fair amount of heat is transferred to it. In the real world, of course, extremely high flow rates would result in heating of the coolant from friction but, as we shall see, we do not need flow rates that high to have minimal coolant rise in going through the chiller. We develop an expression for wort outlet temperature for coolant flow approaching infinity shortly.

In order to calculate performance for a real chiller we would need to know the details of the internal geometry of the chiller in order to calculate the contact area per unit length $W$. We would also have to obtain a value for the thermal conductivity, $G$. Obtaining these values would be tricky because so many subtle factors are involved. We are intersted in the more practical problem of estimating the performance of real chillers and we adopt the approach of making a set of measurements on an existing design to determine its efficiency for the experimental flow rates. We then solve Equation (1.26) for $\alpha L$:

$$\alpha L = -ln\left[\frac{1-\eta}{1-\eta\frac{k_c}{k_w}}\right] \approx -ln\left[\frac{1-\eta}{1-\eta\frac{F_w g}{F_c}}\right] \tag{1.35}$$

with the approximation based upon the assumption that the coolant is water whose specific heat is about the same as the specific heat of wort. The ratio of the density of wort to the density of water is simply the specific gravity, $g$, of the wort so that

$$\frac{k_c}{k_w} = \frac{F_w \rho_w S_w}{F_c \rho_c S_c} \approx \frac{F_w g}{F_c} \tag{1.36}$$

Now

$$\alpha L = (k_w - k_c)L = WGL\left(\frac{1}{F_w \rho_w S_w} - \frac{1}{F_c \rho_c S_c}\right) \approx \frac{WGL}{\rho_c S_c}\left(\frac{1}{F_w g} - \frac{1}{F_c}\right) \tag{1.37}$$

with the approximation based upon the same assumptions. If we define

$$Q = \frac{WGL}{\rho_c S_c} \tag{1.38}$$

then we can obtain an estimate of $Q$, which depends only on the geometry and materials of the chiller from

$$\hat{Q} = \frac{(\alpha L)_{meas}}{\left(\frac{1}{F_w g} - \frac{1}{F_c}\right)} \tag{1.39}$$

Some examples should be illustrative here. A chiller consists of 50 feet of half inch copper refrigeration tubing coaxial within a one inch rubber hose. The coaxial tubes are wound into a coil. When water at 56.5 °F was admitted to this chiller at a flow rate of 290 gph boiling water was cooled to 61°F at a flow rate of 53 gph. The efficiency is (212 - 61)/(212 - 56.5) = .9711. Substitution of these numbers into Equation (1.35) gives $(\alpha L)_{meas}$ = 3.3472. Note that the coolant and "wort" were both water in the test so that $g$ = 1. Use of Equation (1.39) gives 217.07 for an estimate of the $Q$ for this chiller.

Another chiller consists of about 26 feet of 3/8 inch refrigeration tubing formed into a coil. The coil is inserted in a jacket made of PVC pipe. With coolant entering at 56.5 °F at a rate of 309 gph water was cooled from 212 °F to 70 °F when the flow was 25.5 gph. The efficiency is thus (212 - 70)/(212 - 56.5) = 91.318%. Inserting this (and the flow rates) into Equation (1.35) and the result into Equation (1.39) gives 65.7 as an estimate of the $Q$ of this chiller.

Once we have a value for $Q$ for a chiller we can obtain $\alpha L$ for any values of flow rate and wort specific gravity:

$$\alpha L = Q\left(\frac{1}{F_w g} - \frac{1}{F_c}\right) \tag{1.40}$$

and the efficiency is then

$$\eta = \frac{1 - e^{-Q\left(\frac{1}{F_w g} - \frac{1}{F_c}\right)}}{1 - \frac{F_w g}{F_c} e^{-Q\left(\frac{1}{F_w g} - \frac{1}{F_c}\right)}} \tag{1.41}$$

A major goal of this investigation was to come up with some sort of graph or table which would allow a brewer to understand his wort chiller without having to resort to mathematics. It is clear that $\eta$ is easily determined with calculator, spreadsheet or computer program but we would like to be able to obtain approximate performance data without resorting to those tools if possible. The situation looks bad as $\eta$ is a function of 3 variables which makes graphical representations difficult. Salvation comes if we define the variables

$$q_w \equiv \frac{Q}{F_w g} \tag{1.42}$$

and

$$q_c \equiv \frac{Q}{F_c} \tag{1.43}$$

Using these in Equation (1.41) gives

$$\eta = \frac{1 - e^{(q_c - q_w)}}{1 - \frac{q_c}{q_w} e^{(q_c - q_w)}} \tag{1.44}$$

Now we have $\eta$ as a function of 2 parameters and we can solve this new equation for $q_w$ as a function of $q_c$ with $\eta$ as a parameter. We did this by rearanging Equation (1.44)to

$$1 - \eta - e^{(q_c - q_w)}\left(1 - \eta\frac{q_c}{q_w}\right) = 0 \tag{1.45}$$

and using root finding techniques to solve it. The numerical properties of Equation (1.45) are strange in that it is difficult to locate the roots. We chose a range for $q_c$ spanning 0.001 to 100 which results in solutions within this same range. It was necessary to divide the solution range into 100,000 subintervals in order for the Recipes routine ZBRAC to be able to find the roots when $\eta$ was reduced to 80%. Routine RTBIS was used to compute the roots found by ZBRAC.

Figure 1.1 is a plot of data computed as just described with the important modification that the *reciprocals* of the $q$'s are what is plotted. It is hoped that this figure is all that a wort chiller user requires to understand his equipment provided, of course, that his equipment fits within the envelope of the figure. Given a value of $Q$ one is easily able to determine chiller performance for given flow rates or, what is more important, determine appropriate flow rates for a desired level of efficiency. Further the user may determine $Q$ using the curve. We illustrate this first.

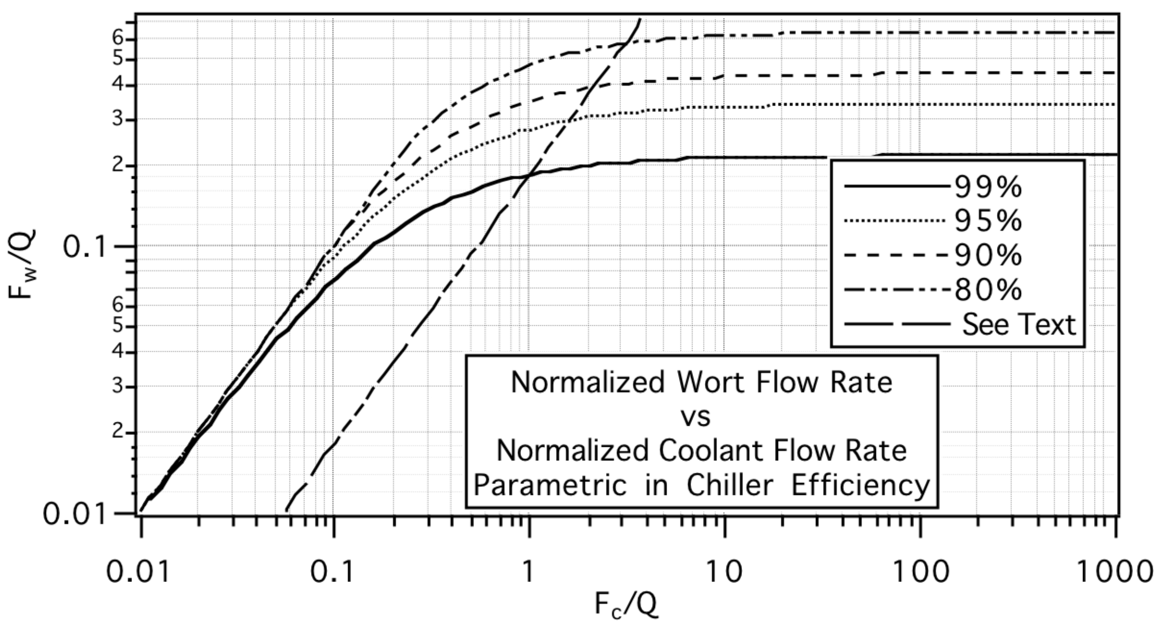


Figure 1.1 Chiller Performance Graph

We previously cited the example of the wort chiller with a $Q$ of 217 as determined from its demonstrated efficiency of 97.11% with wort and coolant flows of respectively 53 gph and 290 gph. The straight line on Figure 1.1 is for $\rho F_W = 0.1827F_c$ with the coefficeint being the ratio of the flow rates. The point at which this line crosses the 97.11% efficiency line gives the values for $\rho F_w/Q$ and $F_c/Q$ which pertained when the measurements were taken. We must eyeball interpolate because the 97.11% line does not appear on the plot. Doing this we get approximately 0.26 for $\rho F_w/Q$ which, divided into the wort flow rate of 53 gph gives 204 as an estimate of $Q$. For $F_c/Q$ we estimate 1.4 which, divided into the coolant flow rate of 290 gph gives a $Q$ estimate of 207. Both of these estimates are quite close to the numerically computed value of 217.

Once a value of $Q$ is determined Figure 1.1 becomes quite informative. We see, for example, that for a given efficiency, increasing the coolant flow rate increases the rate at which wort can be put through the chiller on a one for one basis up to the point where the normalized flow rate gets larger than say 0.1. Beyond 1.0 there is little advantage to increasing the coolant flow any further. If we consider, therefore, an $F_c/Q$ value of 1 a good value we see that the efficiency of the chiller is a strong function of the wort flow rate. Remember that the wort flow must include the wort specific gravity factor.

Example Problem: A brewer wishes to chill a Pilsner of specific gravity 1.055 to 34 °F. He has two chillers like the ones used in the example i.e. they have respective $Q$'s of 217 and 66. His mains water is at 56 °F and the pressure is sufficient to force 290 gph through the larger chiller. He intends to circulate ice water from a bucket through the smaller chiller. He has a pump which is capable of pushing 560 gph through the smaller chiller. How fast can he chill his wort? Note: these numbers are all real.

Solution: The normalized coolant flow in the larger chiller is 290/217 = 1.34 which is a good value. Let us shoot for 99% efficiency and see how fast the wort can flow through the first chiller. The $\rho F_w/Q$ value from the 99% curve on Figure 1.1 for $F_c/Q$=1.34 is approximately 0.19 which when multiplied by 217 gives a flow of 41.2. Dividing by the specific gravity gives 39 gph as the allowable rate for 99% efficiency. As the difference between wort and coolant inlet temperatures is (212 - 56) = 156° the 1% decrement from a perfect chiller will result in wort leaving the chiller at 56 + 1.56 = 57.6°F.

The normalized coolant flow for the smaller chiller is 560/66 = 8.5 which is substantially larger than required. A flow rate of one eighth of this (i.e. 70 gph) would suffice. The two chillers'wort path are in series so that the flow rate inthe cond is the same as in the first i.e. 41.2 (specific gravity included). Normalized by $Q$ this is a $\rho F_w/Q$ value of 41.2/66 = 0.62. This value and the 8.5 value intersect slightly above the 80% efficiency line so that we would probably acheive about 79% i.e. 21% less than perfect. The inlet temperature difference is now (57.6 - 32) = 25.6 and 21% of this is 5.4 ° so that the output temperture from the second chiller would be 37.4 °F. This is higher than desired and so the wort flow must be restricted as we have already indicated that increasing the coolant flow further will have minimal effect. We want a 2° rise above 32° and that is 2/25.6~8% of the inlet temperature difference. Thus we need about 92% efficiency from the second chiller. Moving down the $F_c/Q$=8.5 line we estimate that $\rho F_w/Q$ = 0.4 corresponds to about 92%. Multiplying by $Q$ and dividing by $\rho$ we get a flow of 26 gph.

If we restrict the flow through the chillers to 26 gph the normalized rate into the first is (26)(1.055)/217 = 0.126. This intersects with the normalzed coolant flow ordinate of 1.34 at well below the 99% efficiency line so that we assume the wort output temperature from the first stage to be quite close to the coolant inlet temperature of 56°F. This means that the second stage inlet temperature differentail is probably closer to (56-32)=24 ° and our rise will be 8% of this or 1.9° so that things look good on paper. The actual results should be reasonably close to the calculated ones.